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II - THE COSMIC RAY ANISOTROPY

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A Simple Model of the Interplanetary Magnetic Field II: The Cosmic Ray Anisotropy

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Abstract

The motion of high-energy charged particles in the stretched dipole field of part I is analyzed by a method generally used for motion in the geomagnetic dipole field. An attempt is then made to find out whether the observed anisotropy of cosmic ray intensity may be due to a mechanism similar to that causing the east-west asymmetry near the earth's surface. In the present model this turns out possible, provided that the outer boundary is only about 2 astronomical units from the sun and provided that the direction of the interplanetary field does not reverse when that of the sun's polar field does, as was observed in 1958.

Two other theories of the cosmic ray anisotropy, ascribing it to either a sunward flux density gradient or to the Compton-Getting effect, are also discussed. It is shown that in general both effects occur together; for conservative fields they cancel each other and no anisotropy occurs, as one might indeed expect from Liouville's theorem. Consequently, any gradient of cosmic ray flux density which might be measured in interplanetary space is not necessarily connected with the observed anisotropy.

MOTION OF COSMIC RAY PARTICLES IN A STRETCHED DIPOLE FIELD

The Lagrangian for the motion of a particle with mass m and charge q in an electromagnetic field with magnetic and electric potentials $\underline{A}$ ($\underline{B}$ = curl $\underline{A}$) and Ψ_0 is

$$L = -m_0 c^2 (1 - v^2/c^2)^{\frac{1}{2}} + q(\underline{A} \cdot \underline{v}) - q\Psi_0 \tag{1}$$

If neither $\underline{A}$ nor Ψ_0 depend on the azimuth angle φ, then $\partial L/\partial \dot{\varphi} = \chi_0$ is a constant of motion ("Störmer's first integral"). Denoting by p the momentum of the particle and by ω the angle between it and the φ direction (so that $p_\varphi = p \cos\omega$) the following expression is obtained

$$\chi_0 = r \sin\vartheta \, (p \cos\omega + q A_\varphi) \tag{2}$$

Because of the electric potential Ψ_0, p is not conversed. However, the energies of interest here are considerably larger than the changes they undergo in the electric field so that, to simplify the calculation, p. will be considered as constant. From equation (6a), part I, it follows that

$$A_\varphi = -\frac{\partial \Psi_2}{\partial \vartheta}$$

in region II $\qquad A_\varphi = \frac{3}{2} \frac{\mu_0 m}{4\pi R_0} \frac{\sin\vartheta}{r}$

in region III $\qquad A_\varphi = \frac{3}{2} \frac{\mu_0 m}{4\pi R_0} R_1 \frac{\sin\vartheta}{r^2}$

The trapping properties of the "stretched dipole" field will be investigated in the same manner as has been done for the ordinary dipole (Störmer, 1950; Fermi, 1950). First, let all lengths be measured in momentum-dependent "Störmer units"

$$1_{st.} = \left(\frac{3 q m \mu_0 R_1}{8\pi R_0 p} \right)^{\frac{1}{2}} = (R_0 R_1)^{\frac{1}{2}} \left(\frac{q B_s}{p} R_0 \right)^{\frac{1}{2}} \tag{3}$$

The first factor is a mean value of r in region II, while the second is the ratio between R_0 and the gyration radius at $r = R_0$, $\vartheta = \pi/2$, where the field intensity is B_s (neglecting the toroidal component). Choosing $B_s = 1$ gauss, q as the proton's charge, $R_o = 7.10^{10}$ cm (the solar radius) and measuring p in Bev/c

$$1_{st} = 145\,(R_o R_1/p)^{1/2} \qquad (4)$$

in störmer units, $R_1 < 1$ if

$$R_1/R_o < 21\,000/p \qquad (5)$$

Let (2) be rewritten in störmer units, defining

$$\chi_1 = \chi_o \left(\frac{8\pi R_o}{3\mu_o m R_1 q p} \right)^{1/2}$$

Then, in region II (6a)

$$\cos\omega = \frac{1}{r}\left(\frac{\chi_1}{\sin\vartheta} - \frac{\sin\vartheta}{R_1} \right)$$

and in region III $\cos\omega = \frac{\chi_1}{r\sin\vartheta} - \frac{\sin\vartheta}{r^2}$ (6b)

Consider charged particles with given momentum p and Störmer invariant χ_1. There will in general exist certain parts of the (r, ϑ) plane where equations (6) yield $|\cos\omega| > 1$ and which therefore are not accessible to these particles. The rest of the plane forms the "allowed region", and if this region is multiply connected, trapping may occur.

The allowed region in region II is simply connected. To see this, it is best to consider a single radial direction with fixed ϑ. For all points having this ϑ, the bracketed term in (6a) is constant and $|\cos\omega|$ is a monotonic single-valued function of r. As the origin is approached, $|\cos\omega|$ increases steadily until, for any ϑ, a forbidden region is reached (except for $\vartheta = \arcsin(\chi_1 R_1)^{1/2}$ where the allowed region extends to the origin). Thus the forbidden regions cluster around the origin and the allowed region is simply connected.

The allowed regions of the dipole field in III, on the other hand, are multiply connected (Störmer, 1950; Fermi, 1950) for $\chi_1 > 2$. These regions - with which the allowed regions of II merge smoothly - consist of an inner "trapped" region in which $r < 1$ everywhere and a "free" region where $r > 1$. In the model used here, then, the occurrence of trapping depends on equation (5) being satisfied.

Consider now orbits in the equatorial plane ($\sin\vartheta = 1$) and let every orbit with $\chi_1 < 2$ be called "free" and every one with $\chi_1 > 2$ "trapped". Penumbral effects (Schwartz, 1959) are therefore neglected. If equation (5) is just barely satisfied (e.g. $R_1 = 0.9$) it is easily seen from (6a) that no trapped orbits penetrate very far into region II. As R_1 decreases, this situation changes rapidly until at $R_1 = \frac{1}{2}$, for <u>any</u> r, orbits with $\omega < \pi/2$ are trapped and orbits with $\omega > \pi/2$ are free. Below $R_1 = 1/(1 + \sqrt{2})$, <u>all</u> orbits arriving in II are trapped.

THE ANISOTROPY OF COSMIC RAY INTENSITY

Because of their large gyration radii in the interplanetary field, cosmic ray particles are likely to reflect in their behaviour the gross structure of this field rather than local irregularities. One of the important properties of these particles is their anisotropy, which manifests itself in a solar daily variation of about 0.3% observed on earth. The direction of the maximum flux is approximately tangential to the earth's orbit on the afternoon side and the range of energies at which the anisotropy has been observed is roughly 7 - 20 Bev. In this range, the relative modulation is nearly independent of energy (McCracken, Rao and Venkatesan, 1963) in sharp contrast with other types of cosmic ray intensity variations, which generally decrease rapidly with increasing energy.

It has been often suggested that this anisotropy is caused by an interplanetary dipole field (Janossy, 1937; Alfven, 1947; Dwight, 1950; Elliot, 1960, 1962). The idea is, essentially, that a weak scattering mechanism operates to fill the trapped orbits which, however, are less densely populated than the free ones due to some additional loss mechanism, e.g. scattering into orbits hitting the sun (Elliot, 1960). In the energy range where some of the radiation received on earth is trapped and the rest arrives directly, an anisotropy will be observed, with maximum effect in the ♈ direction. A similar anisotropy ("the east-west effect") occurs in the earth's magnetic field.

In the equatorial plane of the stretched dipole, the anisotropy is most pronounced when $R_1 = \frac{1}{2}$. Assuming that the daily variation indeed arises in this fashion, we choose p = 15 Bev/c and obtain

$$R_1 = 350\ R_o$$

If the solar radius is chosen for R_o, R_1 turns out to be approximately two astronomical units - considerably less than is generally believed, but not impossible (for discussion and references, see analysis by Axford, Dessler and Gottlieb, [1963]).

There are two profound difficulties with this explanation. First, the polar field of the sun was observed to reverse its direction during the solar maximum of 1958 (Babcock, 1959) whereas the cosmic ray anisotropy was not. It has been suggested that the sun's polar field is not the main source of the interplanetary magnetic field and that the latter does not reverse (Elliot, 1960). It is hoped that this point will be resolved in the near future by measurements taken from space probes.

The second difficulty is that according to this explanation, the anisotropy occurs only in a very narrow energy band; it does not explain, for instance, the observation of the daily variation underground (Regener, 1962). It is possible, however, that a more realistic (and less abrupt) model of the outer boundary will resolve this problem.

Two other explanations for the cosmic-ray anisotropy have been advanced, namely that it is caused either by a radial gradient of the cosmic-ray flux or through the Compton-Getting effect. Even without detailed assumptions about the interplanetary magnetic field, these theories run into difficulties connected with Liouville's theorem, as will now be discussed.

THE DENSITY GRADIENT MODEL

This theory has been described by Dattner and Venkatesan (1959) and worked out in detail by Elliot (1960, 1962). One of its basic assumptions is that the interplanetary magnetic field in the vicinity of the earth is perpendicular to the ecliptic; of course, this does not agree with the radial stretching of the magnetic lines of force by the solar wind, but this will not be considered now. Let r be the distance from the sun to an observer on earth and consider particles with momentum p, which will have a gyration radius $a(r)$ in the earth's vicinity. Particles arriving tangentially to the earth's orbit from one direction will then have their guiding center at distance $(r + a)$, while those arriving from the opposite direction will have it at $(r - a)$. If there is a sunward gradient in the flux density Φ (reckoned at the guiding center of the particles it describes) the fluxes in the two directions are not equal and their ratio to the first order in (a/r) is $(1 + \delta)$, where (Elliot, 1960)

$$\delta = \frac{2a}{\Phi} \frac{d\Phi}{dr} \tag{7}$$

There is good reason to believe a density gradient actually exists in interplanetary space, since it has been observed that the flux density near the sun undergoes a modulation connected with the solar cycle, and this modulation presumably extends only a finite distance from the sun. A different question is whether the gradient is pronounced in the vicinity of the earth's orbit. No evidence of an appreciable gradient was found by either Pioneer V (Simpson, Fan and Meyer, 1962) or Mariner II (Anderson, 1963); however, the radiation detectors aboard both these space probes were sensitive down to energies below 100 Mev, so that the absence of a density gradient in the energy range in which an anisotropy is observed on earth may not be considered proven.

A gradient of flux density is not, however, sufficient to create an anisotropy. As a simple illustration, suppose the radiation is acted upon by an electric field due to a positively charged sun. In such a field there will exist a flux density gradient, but because of Liouville's theorem, if the radiation is isotropic far from earth it will remain so anywhere in the field (effects of trapping are not considered now). It is instructive to examine the mechanism by which this happens.

Consider a particle with charge q moving in the symmetry plane of a magnetic dipole field set up around the (positively charged) sun, and assume for simplicity that the motion is nonrelativistic. By Liouville's theorem, with phase-space density τ

$$\Phi = \frac{\tau}{m} p^3$$

$$\frac{1}{\Phi}\frac{d\Phi}{dr} = \frac{3}{p}\frac{dp}{dr}$$

Let W(r) be the mean kinetic energy at distance r and E(r) the (radial) electric field intensity there. Then

$$\frac{dp}{dr} = \frac{m}{p}\frac{dW}{dr} = -\frac{m}{p} q E$$

substituting a = p/qB one obtains

$$\delta = -6E/vB \tag{8}$$

On the other hand, the electric field also causes the guiding center to drift in the direction of the anisotropy with velocity v_D, which by the nonrelativistic guiding center theory is

$$v_D = \left| \frac{E \times B}{B^2} \right| = \frac{E}{B} \tag{9}$$

In the reference frame of its guiding center, a particle spends equal time moving in any direction in its plane of gyration. Given a large number of particles arriving from infinity, an observer moving with this frame sees an isotropic flux. The flux distribution in a frame of reference moving with velocity v_D relative to a frame of reference in which particles arrive isotropically has been calculated (for the extreme relativistic limit) by Compton and Getting (1935). For non-relativistic motion in which v_D is much smaller than the particle velocity v, one finds that the flux in the forward direction increases by a factor $1 + 3(v_D/v)$ while in the backward direction it is diminished by an equal amount. An anisotropy ratio $1 + (6E/vB)$ will therefore arise, completely cancelling out the gradient effect.

More generally, if a density gradient is responsible for the anisotropy, it cannot be caused by a simple potential field, e.g. by Ψ_0 in the model used here. This is expected to hold even for relativistic particles, for Liouville's theorem remains true at relativistic velocities. It is of course possible that there may exist a nonconservative field in the solar system, by which particles gain (Warwick, 1962) or lose (Singer, Laster and Lencheck, 1962) energy. Such a field could, in principle, explain the anisotropy, were it not for the radial stretching of the lines of force. In any case, the solar cycle modulation and any flux density gradient which might be observed in space may very well be due to a conservative mechanism and have no connection with anisotropies.

ANISOTROPY DUE TO THE COMPTON-GETTING EFFECT

A theory has been developed by Ahluwalia and Dessler (1962) ascribing the anisotropy to relative motion between the earth and a frame of reference in which the cosmic radiation is isotropic. The orbital motion of the earth, for instance, would produce such an effect: this will however have an opposite phase to what is observed, and turns out (Dattner and Venkatesan, 1959) to have an amplitude of only 0.03%. In this theory, the sun is assumed to be surrounded by matter flowing radially outwards, as in the model used here. An electric field is then set up, which causes cosmic ray particles to drift across it and be isotropic in a frame of reference moving with the drift velocity. An earlier theory of this kind, by Brunberg and Dattner (1954), assumes the electric field is created by co-rotation of the interplanetary gas with the sun, extending at least to the earth's orbit.

In a highly conducting ionized gas an electric field will indeed exist, tending to the limiting value of $-[\underline{v} \times \underline{B}]$. However, if $\underline{B}$ is axisymmetric around the rotation axis

$$\mathrm{curl}\ \underline{E} = -\frac{\partial \underline{B}}{\partial t} = 0$$

so that $\underline{E}$ is conservative and according to the conclusions of the previous section, no anisotropy arises.

There remains the possibility that the field is not symmetric around the solar rotation axis, e.g. due to "beams" of enhanced velocity as suggested by Alfven (1956). In that case, however, it is hard to explain the constancy of the direction of the anisotropy. Assume the field is increasing at a certain time, creating an anisotropy in the observed direction. Several days later the field will be dropping to its previous value, so that particles in those orbits in which acceleration took place in the first instance will now be decelerated. One would then expect the anisotropy to reverse or at least undergo a considerable change in direction. One would also expect a much better correlation than is observed between the amplitude of the anisotropy and solar disturbances.

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